The problem is examined of motion of a single spherical bubble with mass trans fer in a swirling liquid flow. It is assumed that the bubble rotates without tangential slip while the dissolution process occurs without chemical reaction and heat release. A numerical solution is performed of the system of motion and dissolution equations by the method of prediction and correction. The floating velocity and the change in bubble mass as it moves toward the vortex axis are computed.

The analysis of technological processes associated with gas-liquid phase interac ion is impossible without knowledge of the velocity of gas bubble motion and the mass elinination from them. Clarification of the regularities of the motion and dissolution perm.ts optimization of the constructive solutions of industrial apparatus.

A sufficiently significant number of experimental and theoretical researches [1-4] has been devoted to the regularities of liquid and gas motion during bubbling in a twisted layer. Different aspects of the processes proceeding in apparatus of rotor type are here investigated in practice in all the researches. The hydrodynamics and mass transfer in bubblers of the vortex change type (with tangential delivery of the liquid) have been studied noticeably less [5, 6]. A method is proposed in this paper for the theoretical computation of the velocity of floating and mass elimination from a single spherical gas bubble during bubtling in a twisted layer for apparatus of any kind.

Let us consider interaction of a gas bubble with a twisted weightless liquid flow whose motion is characterized by a known distribution of the tangential $u$ and radial $u_{r}$ velocity components.

Motion of a bubble of constant shape during diffuse dissolution is subject to the second Newton law, which in the case under consideration is written in the form

$$
\begin{equation*}
\left(m+\rho_{\ell} k_{v} V\right) \frac{d^{2} r}{d t^{2}}=\sum_{i} F_{i} \tag{1}
\end{equation*}
$$

The total vector of the forces acting on the bubble consists principally of the following forces

$$
\begin{equation*}
\sum_{i} F_{i}=F_{\mathrm{e}}+F_{\mathrm{d}} \tag{2}
\end{equation*}
$$

where $F_{e}$ is the expulsion force depending on the pressure distribution over the bubble surface, and $F_{d}$ is the drag force caused by bubble motion with respect to the liquid.

The expressions for these forces take the following form in the case under consideration

$$
\begin{gather*}
F_{\mathrm{e}}=\int_{s} P \mathrm{n} d s=-\left(\rho_{\ell}-\rho_{\mathrm{g}}\right) V \frac{u_{\varphi}^{2}}{r}  \tag{3}\\
F_{\mathrm{d}}=\frac{\rho_{\ell} \pi D^{2} k_{d}\left(W_{r}-u_{r}\right)^{2}}{8}
\end{gather*}
$$

[^0]Substituting (2) and (3) into (1) and manipulating, and taking into account that $W_{r}=d r / d t$ we obtain

$$
\frac{d W_{r}}{d t}=-\frac{\left(\rho_{\ell}-\rho_{\mathrm{g}}\right) u_{\varphi}^{2}}{\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{v}\right) r}+\frac{3 \rho_{\ell} k_{d}\left(W_{r}-u_{r}\right)^{2}}{4\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{0}\right) D}
$$

Going over the variable $t$ to the variable $r$ in the equation of motion, we perform the following manipulations

$$
\frac{d W_{r}}{d t}=\frac{d W_{r}}{d r} \frac{d r}{d t}=\frac{d W_{r}}{d r} W_{r}=\frac{d}{d r}\left(\frac{W_{r}^{2}}{2}\right)
$$

whereupon we obtain the equation of motion of the gas bubble

$$
\begin{equation*}
\frac{d E}{d r}=-\frac{2\left(\rho_{\ell}-\rho_{g}\right) u_{\varphi}^{2}}{\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{0}\right) r}+\frac{3 \rho_{\ell} k_{d}\left(\sqrt{E}-u_{r}\right)^{2}}{2\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{v}\right) D} \tag{4}
\end{equation*}
$$

where $E=W_{r}{ }^{2}$.
The bubble mass can be determined from the equation of state of the gas bubble

$$
\begin{equation*}
m=\frac{P V M}{R T}=\frac{\pi D^{3} P M}{6 R T} \tag{5}
\end{equation*}
$$

We find the change in bubble mass along the radius by differentiating (5) with respect to $r$ while taking into account that $T=$ const

$$
\begin{equation*}
\frac{d m}{d r}=\frac{\pi M}{6 R T}\left[D^{3} \frac{d P}{d r}+3 D^{2} P \frac{d D}{d r}\right] \tag{6}
\end{equation*}
$$

On other other hand, the intensity of bubble dissolution can be represented in the form

$$
\begin{equation*}
\frac{d m}{d r}=\frac{I(r)}{W_{r}} \tag{7}
\end{equation*}
$$

where $I=d m / d t$ is the intensity of mass elimination from the bubble. Equating the right sides of (6) and (7), we arrive at an equation describing the change in bubble diameter as it floats to the axis of rotation

$$
\begin{equation*}
\frac{d D}{d r}=\frac{2 R T I}{\pi M \sqrt{\bar{E} D^{2} P}}-\frac{D \frac{d P}{d r}}{3 P} \tag{8}
\end{equation*}
$$

In the case of intensive rotation $\left(u_{q} \gg u_{r}\right)$ the functions $d P / d r$ and $P$ in (8) have the form

$$
\begin{equation*}
\frac{d P}{d r}=\frac{\rho_{\ell} u_{\varphi}^{2}}{r} ; \quad P=\mathrm{const}+\rho_{\ell} \int \frac{u_{\varphi}^{2}}{r} d r \tag{9}
\end{equation*}
$$

Therefore, by substituting (9) into (8) we finally obtain a system of differential equations that describes the motion of a gas bubble being dissolved in a twisted liquid flow

$$
\begin{gather*}
\frac{d E}{d r}=-\frac{2\left(\rho_{\ell}-\rho_{\mathrm{g}}\right) u_{\varphi}^{2}}{\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{v}\right) r}+\frac{3 \rho_{\ell} k_{d}\left(\sqrt{E}-u_{r}\right)^{2}}{2\left(\rho_{\mathrm{g}}+\rho_{\ell} k_{v}\right) D}  \tag{10}\\
\frac{d D}{d r}=\frac{2 R T I}{\pi M \sqrt{E} D^{2} P}-\frac{\rho_{\ell} D u_{\varphi}^{2}}{3 r P}
\end{gather*}
$$

The initial conditions of the problem are here written as follows

$$
D\left(r_{1}\right)=D_{0} ; E\left(r_{1}\right)=u_{r}^{2}\left(r_{1}\right)
$$

The system (10) was solved as an example by the numerical method of prediction and correction [7] for the case of motion and dissolution of a Stokes bubble (the expression for the function $I$ is known from [8]):

$$
I=-2 \sqrt{\sqrt{x}}\left[D_{f}\left(\sqrt{E}-u_{r}\right)\right]^{1 / 2} D^{3 / 2}\left(c_{p}-c_{\infty}\right)
$$



Fig. 1. Profiles of a bubble radial floating velocity: 1) $r_{1}=0.15 \mathrm{~m}$; 2) 0.2 ; 3) 0.25 .
Fig. 2. Relative change in bubble mass for different chamber sizes: 1) $r_{1}=0.15 \mathrm{~m}$; 2) 0.2 ; 3) 0.25 ; a) $r_{0}=0.03$ m ; b) 0.015 .

The results of calculations performed for different vortex chamber sizes are presented in Figs. 1 and 2. It was assumed in all the computational cases that the liquid mass flow rate and the pressure are constant while the radial and tangential velocity distributions have the form

$$
\begin{gathered}
u_{r}=\left\{\begin{array}{l}
\frac{Q r}{2 \pi r_{0}^{2}} ; r \leqslant r_{0} \\
\frac{Q}{2 \pi r} ; r \geqslant r_{0}, \\
u_{\varphi}=\frac{\Gamma}{r} .
\end{array}\right.
\end{gathered}
$$

The distribution of the bubble floating velocity along the radius of the vortex chamber is shown in Fig. 1. Analysis of the dependences obtained permitted making the deduction that the floating velocity profile for constant mass flow rate and pressure on the chamber axis is independent of the vortex chamber diameter. A change in the chamber diameter results in changes in the floating velocity profile in the acceleration section. For a constant flow swirl intensity $\Gamma$ the changes in the chamber outer diameter results in flattening of the profile in this section.

The dependence of the relative change in bubble mass along the radius is represented in Fig. 2 for different central hole diameters and chamber diameters. It is seen from the figure that the mass elimination depends substantially on the quantity ( $r_{1}-r_{0}$ ) when the rotational velocity profile is constant. The mass elimination intensity drops with the approach to the chamber axis because of the increase in the radial floating velocity. Diminution of the inner diameter of the central hole results in diminution of the mass elimination since the floating velocity grows here.

In particular, the investigation performed permits making a practical deduction that it is expedient to manufacture a bubbler in the form of a vortex chamber with low height and possibly large diameter to raise the degree of gas dissolution in a liquid without substantial change in the gas bubble motion.

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LASER PLASMA IN A VACUUM AS AN INTENSE SOURCE OF UV
RADIATION
A. P. Golub' and I. V. Nemchinov

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By using estimates and numerical simulation it is shown that an aluminum plasma sustained by $\mathrm{CO}_{2}$-laser radiation can radiate intensively in a vacuum in the hard ultraviolet range. High values of the conversion coefficient are achieved for lower radiation flux densities than in the neodymium laser case.

A plasma initiated by a "burst" of absorption [1-3] into disintegrating erosion vapors occurs on the surface of an opaque obstacle in a vacuum under the action of IR band laser radiation. If further action is realized under plane one-dimensional motion conditions, then as the geometric and optical thicknesses of the plasma layer increases a considerable part of the delivered laser energy can be converted into thermal radiation energy emitted by a plasma in a vacuum [4-6]. In that case, when the geometric thickness of the vapor layer becomes commensurate with the characteristic dimension of the spot being irradiated, a selfconsistent vapor scattering and heating mode is set up [7-9] because of expansion in the side direction, where the gasdynamic and radiation parameters achievable at the end of the plane stage are maintained by laser radiation at a quasistationary level. Estimates [9-11] and experiments [10-13] show that in this case reradiation can also be a sufficiently substantial factor. Among the distinctive features of a laser plasma in a vacuum should be the possibility of heating it to high enough temperatures to assure emission of radiation of the necessary hardness down to the far vacuum ultraviolet and soft x-ray. The efficiency of plasma heating is increases as the energy of the laser quanta diminishes. Meanwhile, the optical thickness of the plasma heated in a self-consistent mode drops for its intrinsic thermal radiation. It is interesting to investigate how the radiation properties of a laser plasma change here.

The thermal radiation of a plasma was examined in [4-6, 9-13] just for the Nd-laser case. By using estimates and numerical computations the reradiation of an aluminum plasma with characteristic dimension of approximately 1 cm sustained by $\mathrm{CO}_{2}$-laser radiation at moderate flux densities, namely, $10-10^{4} \mathrm{MW} / \mathrm{cm}^{2}$, is investigated in this paper. A comparison is also made with the case of Nd-laser radiation action. Lateral broadening of the plasma jet is simulated by spherically symmetric motion.

1. The plasma energy losses due to its intrinsic thermal radiation are determined by the temperatures achieved therein, the densities, and the characteristic dimensions of the plasma.
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